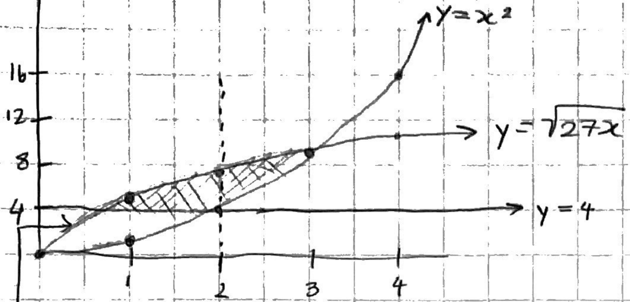
Area between curves

① Estimate the area of the lake

$(65+45+50+80+70)40$

Area $= 12400 \text{ ft}^2$

② Sketch the graphs of $y=4$, $y=x^2$, and $y=\sqrt{27x}$



③ Shade the triangular region bounded by the graphs of the 3 functions

④ Compute x-coordinate of left endpt of region.

$y=4, \quad y=\sqrt{27x}$

$\sqrt{27x}=4$

$27x=16$

$\boxed{x=\frac{16}{27}}$

⑤ Compute x-coordinate of right endpoint of region.

$y=x^2, \quad y=\sqrt{27x}$

$\sqrt{27x}=x^2$

$27x=x^4$

$\frac{27x}{x^4}=1$

$\frac{27}{x^3}=1$

$x^3=27$

$\boxed{x=3}$

⑥ Find the x-coordinate where the two bottom curves meet.

$y=4, \quad y=x^2$

$x^2=4$

$\boxed{x=2}$

⑦ Sketch a vertical line at the x coordinate you found in the last problem

⑧ Compute area of left subregion

$\int_{16/27}^{2} \sqrt{27x}\,dx - \int_{16/27}^{2} 4$

$=\sqrt{27}\int_{16/27}^{2} \sqrt{x}\,dx \qquad - 4x\Big|_{16/27}^{2}$

$=\sqrt{27}\cdot \frac{2}{3} x^{3/2}\Big]_{16/27}^{2}$

$=(\sqrt{27}\cdot \frac{2}{3}(2)^{3/2}) - (\sqrt{27}\cdot\frac{2}{3}(\frac{16}{27})^{3/2}) - (4(2)-4(\frac{16}{27}))$

$=(9.79796 - 1.580247) - (8 - 2.37037)$

$= 8.21766 - 5.62963$

$\boxed{= 2.58803}$

⑨ Right subregion: $\int_2^3 \sqrt{27x}\,dx - \int_2^3 x^2\,dx$

$=\sqrt{27}\cdot\frac{2}{3}x^{3/2}\Big]_2^3 - \left(\frac{x^3}{3}\right)\Big]_2^3$

$=(18-9.79795) - (9-2.66667)$

$\boxed{= 1.8675}$

Total area: 4.45678

(16) Recompute area by solving for x as a function of y, then integrate with respect to y. Is this easier? $\Rightarrow$ Yes this is a much simpler method.

$y=\sqrt{27x}$, $y=x^2$

$y^2=27x$ $\boxed{x=\sqrt{y}}$

$\boxed{x=\frac{y^2}{27}}$ ← functions, bounds: 4 to 9

$\int_4^9 \sqrt{y}\,dy - \int_4^9 \frac{y^2}{27}\,dy$

$= \frac{2}{3}y^{3/2}\Big]_4^9 - \frac{1}{27}\cdot\frac{y^3}{3}\Big]_4^9$

$=(18-5.333)-(9-0.79)$

$\boxed{=4.457}$ ← Total Area

① Find the derivative of: $\int_{x^2}^{e^{2x}} (\ln t + \sin t)\,dt$

$\ln t + \sin t \Big]_{x^2}^{e^{2x}} \frac{d}{dx}$

$((\ln(e^{2x})+\sin(e^{2x}))\,2e^{2x}) - (\ln(x^2)+\sin(x^2))\,2x$

$\boxed{=(2e^{2x}\ln(e^{2x})+2e^{2x}\sin(e^{2x}))-(2x\ln(x^2)+2x\sin(x^2))}$

② Evaluate $\int_0^{\sqrt{\pi/2}} x\cos(x^2)\,dx$

$u=x^2 \quad du=(2x\,du\,dx)\frac{1}{2}$

$=\int_0^{\sqrt{\pi/2}} \frac{1}{2}\cos(u)\,du$

$=\frac{1}{2}\int_0^{\sqrt{\pi/2}} \cos(u)\,du$

$=\frac{1}{2}(\sin(u))\Big]_0^{\sqrt{\pi}/2}$

$=\frac{1}{2}(\sin(\sqrt{\pi}/2)) - (\frac{1}{2}(\sin(0)))$

$=0.707106781 = \boxed{\frac{\sqrt{2}}{2}}$

③ $\int x\sqrt{x+1}\,dx$

$u=x+1 \quad du=1\,dx$

$x=u-1$

$\int(u-1)\sqrt{u}\,du$

$=\int u\sqrt{u}-\sqrt{u}$

$=\int u\sqrt{u}-\int\sqrt{u}$